

# **Coherent Effects in the Wave Chaos and in the Chaotic Transmission of Waves**

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## **LONG-TERM GOALS**

Our goal is to develop an analytical and numerical approach for description of chaotic sound wave fields at long range propagation in the ocean on the basis of the most advanced methods of dynamics that include ray dynamics, wave dynamics reconstruction on the basis of ray dynamics, specific asymptotic solutions in the short-wave approximation, and specific diagnostic codes adjusted to the wave-chaos analysis.

## **OBJECTIVES**

Long-range sound propagation in the ocean can be studied using contemporary methods of nonlinear dynamics, resonance theory, theory of chaos and, particularly, quantum chaos. Our research is directed towards developing and understanding of new phenomenon: wave-chaos. The results can be applied to wave propagation in plasma layers and toroidal plasma devices, to mesoscopic systems (electron transport and conductivity of quantum wires and dots).

## **APPROACH**

In our research we use:

1. Geometrical optics in terms of the Hamiltonian formalism
2. Method of parabolic equation
3. Topological analysis of chaotic orbits in the phase space
4. Kinetic theory and fractional kinetics
5. High-performance simulations

## **WORK COMPLETED**

We demonstrated how the chaotic ray motion reveals itself in the sound energy distribution over a time-depth plane. We have shown that the coexistence of chaotic and regular rays may cause an appearance of a “bright spot” in this plane. We analyzed range variations of mode amplitudes and demonstrated that even under conditions of ray chaos the modal structure of the wave field may exhibit surprisingly regular features. We have started a new theme on the chaos – assisted detection of objects. First results have been obtained that show strong changes in the arrival front of acoustic waves, sensitivity of time arrivals to the location of an object.

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## RESULTS

We considered sound-wave propagation in a strongly idealized model of the deep-water acoustic waveguide with periodic range dependence. It is investigated how the phenomenon of ray and wave chaos affects the sound scattering at a strong mesoscale inhomogeneity of the refractive index caused by the synoptic eddy. Methods derived in the theory of dynamical and quantum chaos, are applied. When studying the properties of wave chaos we decompose the wave field into a sum of Floquet modes analogous to quantum states with fixed quasienergies. It is demonstrated numerically that the "stable islands" from the phase portrait of the ray system reveal themselves in the coarse-grained Wigner functions of individual Floquet modes. A perturbation theory has been derived which gives an insight into the role of the mode-medium resonance in the formation of Floquet modes. It is shown that the presence of a weak internal-wave-induced perturbation, giving rise to ray and wave chaos, strongly increases the sensitivity of the monochromatic wave field to an appearance of the eddy. To investigate the sensitivity of the transient wave field we have considered variations of the ray travel times-arrival times of sound pulses coming to the receiver through individual ray paths-caused by the eddy. It turns out that even under conditions of ray chaos these variations are relatively predictable. This result suggests that the influence of chaotic-ray motion may be partially suppressed by using pulse signals. However, the relative predictability of travel time variations caused by a large-scale inhomogeneity is not a general property of the ray chaos. This statement is illustrated numerically by considering an inhomogeneity in the form of a perfectly reflecting bar [1].

We considered wave propagation in a model of a deep ocean acoustic wave guide with periodic range dependence. It is assumed that the wave field is governed by the parabolic equation. Formally the mathematical model of the wave guide coincides with that of a quantum system with time-dependent Hamiltonian. From the analysis of Floquet modes of the wave guide it is shown that there exists a "scarring" effect similar to that observed in quantum systems. It turns out that the segments of an unstable periodic ray trajectory may be distinguished in the spatial distribution of the wave field intensity at a finite wavelength. Besides the scarring effect, it is found that the so-called "stable islands" in the phase space of ray dynamics reveal themselves in the coarse-grained Wigner functions of the Floquet modes [2].

## IMPACT/APPLICATIONS

Our results contribute to general theory of wave propagation in waveguides. They also provide theoretical background for developing methods of acoustic thermometry of the ocean climate.

## REFERENCES

1. "Chaos-induced intensification of wave scattering", by I.P. Smirnov, A.L. Virovlyansky, M. Edelman, and G.M. Zaslavsky (published in Phys. Rev. E, Vol. 72, Art. No. 036204 (2005))
2. "Manifestation of scarring in a driven system with wave chaos", by A.L. Virovlyansky and G.M. Zaslavsky (published in Chaos, Vol. 15, Art. No. 023301 (2005))

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